RD and Instrumental Variables

EDLD 650: Week 5

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Agenda

1. Roadmap (9:00–9:05)

2. Holden paper and DARE #2 (9:05–10:20)

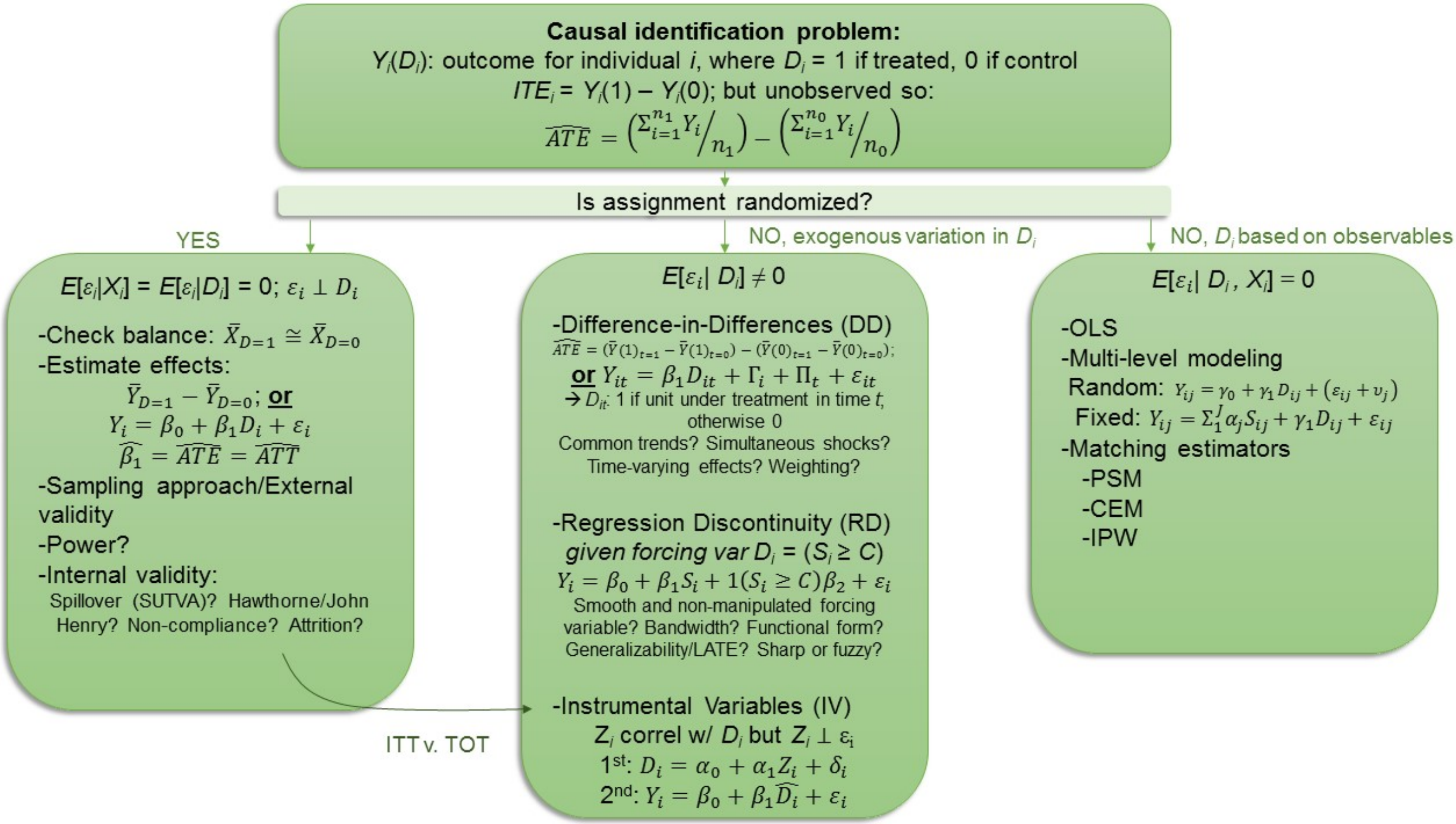
   - Discussion questions
   - DARE debrief

3. Break (10:20–10:35)

4. Instrumental variables (10:35–11:40)

5. Wrap-up (11:40–11:50)

Roadmap



Causal identification problem:
$Y_i(D_i)$: outcome for individual i, where D_i = 1 if treated, 0 if control
$ITE_i = Y_i(1) - Y_i(0)$; but unobserved so:
$\widehat{ATE} = \left(\sum_{i=1}^{n_1} Y_i / n_1\right) - \left(\sum_{i=1}^{n_0} Y_i / n_0\right)$
Is assignment randomized?
YES
NO, exogenous variation in D_i
NO, D_i based on observables
$E[\varepsilon_i|X_i] = E[\varepsilon_i|D_i] = 0$; $\varepsilon_i \perp D_i$
-Check balance: $\bar{X}_{D=1} \cong \bar{X}_{D=0}$
-Estimate effects:
$\bar{Y}_{D=1} - \bar{Y}_{D=0}$; or
$Y_i = \beta_0 + \beta_1 D_i + \varepsilon_i$
$\widehat{\beta_1} = \widehat{ATE} = \widehat{ATT}$
-Sampling approach/External validity
-Power?
-Internal validity:
Spillover (SUTVA)? Hawthorne/John Henry? Non-compliance? Attrition?
ITT v. TOT
$E[\varepsilon_i| D_i] \neq 0$
-Difference-in-Differences (DD)
$\widehat{ATE} = (\bar{Y}(1)_{t=1} - \bar{Y}(1)_{t=0}) - (\bar{Y}(0)_{t=1} - \bar{Y}(0)_{t=0})$;
or $Y_{it} = \beta_1 D_{it} + \Gamma_i + \Pi_t + \varepsilon_{it}$
→ D_{it}: 1 if unit under treatment in time t, otherwise 0
Common trends? Simultaneous shocks? Time-varying effects? Weighting?
-Regression Discontinuity (RD)
given forcing var $D_i = (S_i \geq C)$
$Y_i = \beta_0 + \beta_1 S_i + 1(S_i \geq C)\beta_2 + \varepsilon_i$
Smooth and non-manipulated forcing variable? Bandwidth? Functional form? Generalizability/LATE? Sharp or fuzzy?
-Instrumental Variables (IV)
Z_i correl w/ D_i but $Z_i \perp \varepsilon_i$
1st: $D_i = \alpha_0 + \alpha_1 Z_i + \delta_i$
2nd: $Y_i = \beta_0 + \beta_1 \widehat{D_i} + \varepsilon_i$
$E[\varepsilon_i| D_i, X_i] = 0$
-OLS
-Multi-level modeling
Random: $Y_{ij} = \gamma_0 + \gamma_1 D_{ij} + (\varepsilon_{ij} + \upsilon_j)$
Fixed: $Y_{ij} = \sum_1^J \alpha_j S_{ij} + \gamma_1 D_{ij} + \varepsilon_{ij}$
-Matching estimators
-PSM
-CEM
-IPW

Goals

1. Conduct and interpret RD analysis in simplified data

2. Assess the basic assumptions of the RD design

3. Describe the conceptual and simple mathematical approach for identifying causal effects using the instrumental variables approach

Class 5 Discussion Questions

You DARE-devils!

Break

Instrumental variables

The set-up

Consider the following relationship we would like to estimate:

The effect of a treatment (D_i) on an outcome of interest (Y_i):

$$Y_i = \beta_0 + \beta_1 D_i + \varepsilon_i$$

Now, let's apply this to a particular context:

$$INCOME_i = \beta_0 + \beta_1 COLLEGE_i + \varepsilon_i$$

Don't be so crass!

$$VOTE_i = \beta_0 + \beta_1 COLLEGE_i + \mathbf{X}_i \theta + \varepsilon_i$$

> Describe to your neighbor using the language of causal inference (*omitted variable bias, endogenous, causality, selection bias*) what is wrong with fitting this last regression in a nationally representative sample of adults for which we have records of their voting participation, highest level of education and rich demographic covariates $(\mathbf{X}_i)$.

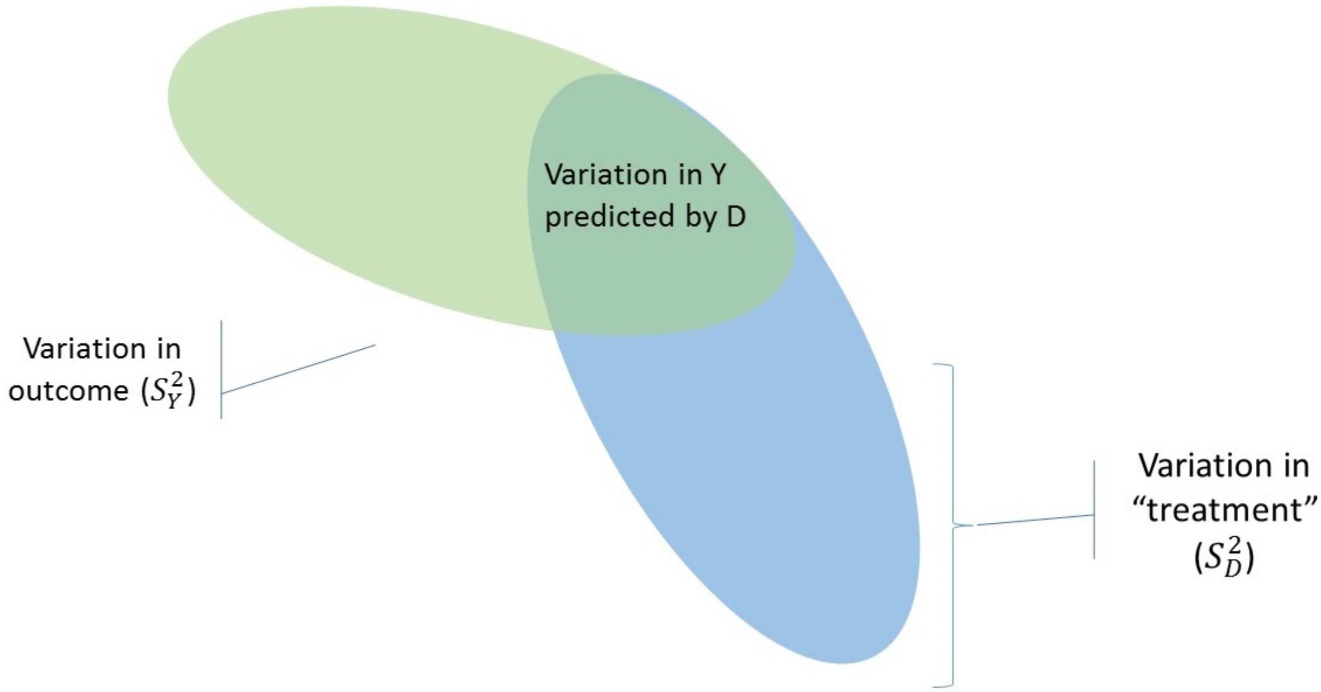
A mysterious solution

Can we fix this without a sudden change across time and geography that we might term a "natural" experiment?

- What if we have data on another mysterious variable...?
- Let's call this variable an "instrument" and assign it the letter Z_i for each individual i
- Let's suppose that it predicts treatment (D_i) and is itself exogenously determined

What would this mean???

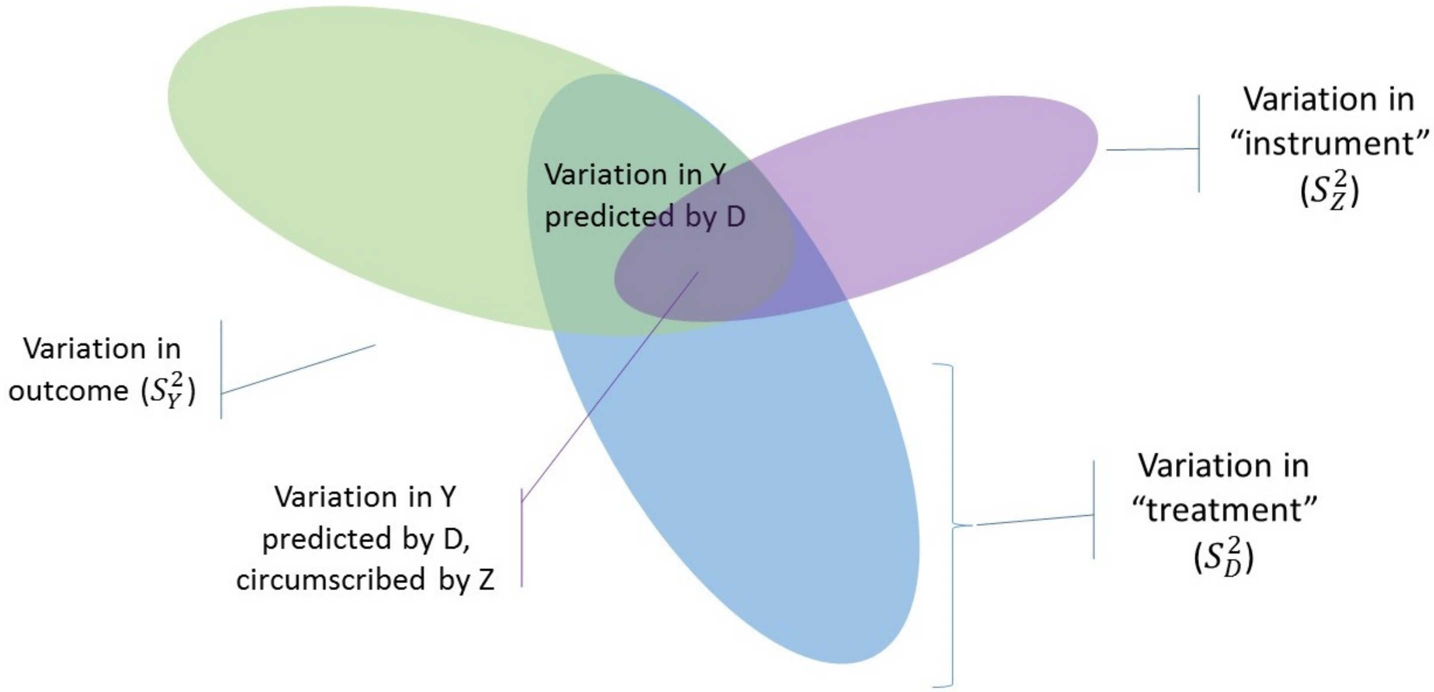
A mysterious "instrument"



OLS estimate: ratio of the area of *overlap of* Y *and* D to the *total area of* D:

$$\hat{\beta}_1^{OLS} = \frac{S_{YD}}{S_D^2}$$

A mysterious "instrument"



IV estimate: ratio of area of *overlap of Y and Z* to area of *overlap of D and Z*. Depends entirely on variation in Z that predicts variation in Y and D:

$$\hat{\beta}_1^{IVE} = \frac{S_{YD}}{S_{DZ}}$$

a Local Average Treatment Effect

An instrument?

But what serves as a helpful and valid instrument?

Valid instruments:

1. Instrument (Z_i) must be correlated with treatment (D_i), *but*
2. Instrument (Z_i) must be orthogonal $(\perp)$ to all other determinants of the outcome (Y_i)
    - Another way of saying it must be uncorrelated with the residuals (ε_i)
3. Instrument must be related to the outcome *only* through the treatment
    - This is known as the exclusion restriction (we'll come back to this)

Can you think of things that might serve as good instruments in the example of college attendance ("treatment") and voting ("outcome")?

Two-stage least squares (2SLS) IV

1st stage:

Regress endogenous treatment (D_i) on instrumental variable (Z_i) using OLS:

$$D_i = \alpha_0 + \alpha_1 Z_i + \nu_i$$

Obtain the *predicted values* of the treatment $(\hat{D}_i)$ from this fit.[1]

2nd stage:

Regress outcome (Y_i) on predicted values of treatment $(\hat{D}_i)$ using OLS:

$$Y_i = \beta_0 + \beta_1 \hat{D}_i + \varepsilon_i$$

[1] This doesn't get the standard errors correct, have to adjust *post-hoc*, but this is automated in all statistical software.

IV assumptions (re-stated)

Stage 1: $D_i = \alpha_0 + \alpha_1 Z_i + \nu_i$

Stage 2: $Y_i = \beta_0 + \beta_1 \hat{D}_i + \varepsilon_i$

Assumptions:

1. Instrument must be correlated with the endogenous predictor (i.e., cannot be a **"weak" instrument**)
2. Instrument must *not* be correlated with the residuals in the first stage equation $(\sigma_{Z\nu} = 0)$
3. Instrument must *not* be correlated with the residuals in the second stage equation $(\sigma_{Z\varepsilon} = 0)$

IV assumptions

Assumptions:

1. Instrument must be correlated with the endogenous predictor (i.e., cannot be a **"weak" instrument**)
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3. Instrument must *not* be correlated with the residuals in the second stage equation ($\sigma_{Z\varepsilon} = 0$)

Problems with #1:

- If Z does not predict D, it would be a **"weak instrument"**
- There would be no (minimal) variation in the obtained predicted values of the question predictor in the second stage
- The estimated regression slope would be *indeterminate* (or close to it)

IV assumptions

Assumptions:

1. Instrument must be correlated with the endogenous predictor (i.e., cannot be a **"weak" instrument**)
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Problems with #2:

- If Z is correlated with ν_i, then Z would be endogenous in the first stage equation
- The values of the question predictor would be replaced by biased predicted values, and the estimated regression coefficient would be biased in ways similar to biased multi-variate regression models

IV assumptions

Assumptions:

1. Instrument must be correlated with the endogenous predictor (i.e., cannot be a **"weak" instrument**)
2. Instrument must *not* be correlated with the residuals in the first stage equation $(\sigma_{Z\nu} = 0)$
3. Instrument must *not* be correlated with the residuals in the second stage equation $(\sigma_{Z\varepsilon} = 0)$

Problems with #3:

The statistical basis for IV of a potentially endogenous relationship is that:

$$\beta_1 = \left(\frac{\sigma_{YZ}}{\sigma_{DZ}}\right) - \left(\frac{\sigma_{\varepsilon Z}}{\sigma_{DZ}}\right)$$

where σ_{YZ} is the population covariance of outcome Y and instrument Z, $\sigma_{\varepsilon Z}$ is the population covariation of residual ε and instrument Z, and σ_{DZ} is the population covariance of treatment D and instrument Z.

IV assumptions

Assumptions:

1. Instrument must be correlated with the endogenous predictor (i.e., cannot be a **"weak" instrument**)
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Problems with #3:

The statistical basis for IV of a potentially endogenous relationship is that:

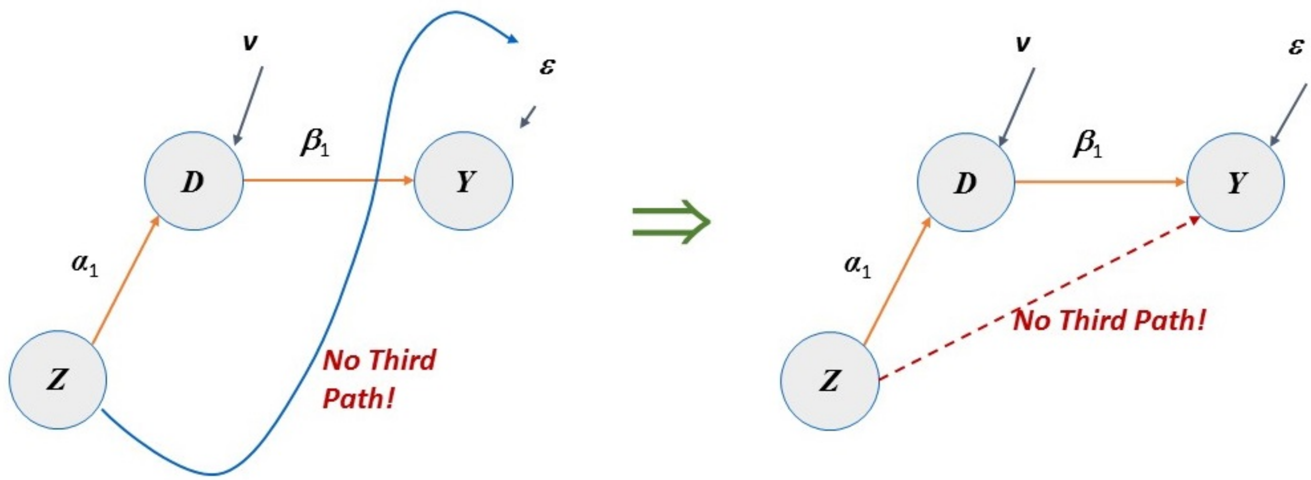
$$\beta_1 = \left(\frac{\sigma_{YZ}}{\sigma_{DZ}}\right) - \left(\frac{\sigma_{\varepsilon Z}}{\sigma_{DZ}}\right)$$

as long as: (1) $\sigma_{\varepsilon Z} = 0$; and (2) $\sigma_{DZ} \neq 0$.

If Z correlated with ε, then $\sigma_{\varepsilon Z} \neq 0$ and β_1 will be biased.

Exclusion restriction (visually)

Those DAG-gone things come back...



The exclusion restriction states that the path by which the instrument influences the outcome goes **exclusively** through the endogenous predictor.

Instrument examples

Outcome	Treatment	Omitted variables	Instrument
Health	Smoking cigarettes	Other neg health behaviors	Tobacco taxes
Labor market success	Assimilation	Ability; motivation	Scrabble score of name
Crime rate	Patrol hours	# of criminals	Election cycles
Female labor market	Number of children	Family preferences; religiosity	First two children same-sex; twin births
Conflicts	Economic growth	Simultaneous causality	Rainfall 😄

Can you think of a good instrument in the example of college and voting (**turn and talk**)? *Hint: try to find something that exogenously predicts college attendance but is unrelated to voting.*

"A necessary but not a sufficient condition for having an instrument that can satisfy the exclusion restriction is if people are confused when you tell them about the instrument's relationship to the outcome." (Cunningham, 2021, p. 321)

Very special instruments

Outcome	Treatment	Omitted variables	Instrument
Test scores	Voucher-based private school attendance	Non-compliance	Original NYSP lottery
Reading achievement	Class size	Incomplete compliance to Maimonides' Rule	Rule-based assignment
Reading achievement	Randomly assigned reading intervention	Incomplete compliance; attrition	Assignment to intervention

All instances in which assignment to treatment is *as-good-as random* but there is imperfect compliance.

NY vouchers (Ch. 4 *MM*)

Recall the NY Voucher experiment from Week 1

- Low-income families *randomized* by lottery to **treatment** and **control** groups
  - Treatment families *received a voucher* to cover partial tuition costs at private schools;
  - Control families *received no voucher*
- Subsequent academic achievement measured for participating children:

$$READ_i = \beta_0 + \beta_1 VOUCHER_i + \mathbf{X}_i \gamma + \varepsilon_i$$

- β_1 represents the causal effect of **voucher receipt** on reading achievement
- Because children were randomly assigned to the experimental conditions, predictor $VOUCHER_i$ is exogenous and children in the “Voucher” and “No Voucher” conditions are equal in expectation, prior to treatment.
- We can obtain an unbiased estimate of β_1 straightforwardly, using OLS regression analysis.

But wait a minute...

While families were randomly assigned to "Voucher" and "No Voucher" conditions, **actual attendance** at private versus public schools was *not* randomly assigned.

In first year of experiment, **5% of kids** whose families *did not receive vouchers* went to private school anyway

- Families wanted their kids taught in a religious setting
- Families wanted their kids out of public schools
- Families had greater financial and social resources

In first year of experiment, **18% of kids** whose families *received vouchers* still went to public school

- Families lived too far from school and couldn't transport
- Families couldn't make up difference of private school fees
- Families didn't feel welcome in private school setting

As a result of these unobserved choices:

1. **Attendance** at public and private school was *not* assigned exogenously
2. Children who **attended** each kind of school were **not** equal in expectation beforehand

Same ole' endogeneity problem

Want to estimate:

$$READ_i = \beta_0 + \beta_1 PRIVATE_i + \mathbf{X}_i \gamma + \varepsilon_i$$

but, unobserved characteristics (such as school accessibility, family resources, motivation, etc.) may determine whether the child goes to private school and also determine her outcomes.

Because these **unobserved characteristics** are omitted as explicit predictors but affect the outcome, their effects are present in the residual (ε_i).

Consequently, $PRIVATE_i$ will be correlated with the residuals and an OLS estimate of β_1 will be biased!

An IV solution!

- Offer of voucher *was* randomized and exogenous
- Offer of voucher likely to be correlated with **attendance** at a private or a public school because many families who got the voucher did in fact use it, and many families who didn't sent their kids to public school
- Being randomized to voucher receipt is unlikely to predict the child's ultimate achievement, *except through its impact on private school attendance*
  - No third path!

We can use instrumental variable estimation, with attendance at private school $(PRIVATE_i)$ as the endogenous question predictor and lottery-based receipt of a voucher $(VOUCHER_i)$ as the instrument!

An IV solution!

We can use instrumental variable estimation, with attendance at private school $(PRIVATE_i)$ as the endogenous question predictor and lottery-based receipt of a voucher $(VOUCHER_i)$ as the instrument!

How would you write this?

1st stage:

$$PRIVATE_i = \alpha_0 + \alpha_1 VOUCHER_i + \alpha_2 READ_i^{pre} + \delta_i$$

2nd stage:

$$READ_i^{post} = \beta_0 + \beta_1 PRIV\hat{A}TE_i + \beta_2 READ_i^{pre} + \varepsilon_i$$

Note the inclusion of baseline reading scores $(READ_i^{pre})$ to improve precision and the inclusion of all covariates from Stage 1 in Stage 2!

β_1 is our causal parameter of interest and represents the estimated Local Average Treatment Effect (LATE) of attending private school on lagged-score adjusted reading scores. *But to whom is it "local"? To whom do these estimates pertain?*

On whom does IV depend?

This is a helpful tool for any causal analysis that relies on an original randomized offer, but is followed by endogenous “take-up.” It also provides considerable insight into what an IV estimator is actually estimating:

	Never takers	Always takers	Compliers
	(never accept treatment)	*(always seek out and obtain treatment)*	*(accept treatment if assigned; accept control if assigned)*
Voucher=1	"Not treated"	"Treated"	"Treated"
Voucher=0	"Not treated"	"Treated"	"Not treated"

An IV estimate is often referred to as the effect of the **Treatement on the Treated (TOT)**. Covariance algebra tells us that only the **compliers** actually contribute to the IV estimate. IV is a LATE estimator, so only those participants whose actions respond to the instrument will participate in the estimate. **IV is the treatment effect for the compliers.**[1]

This approach assumes that there are no **defiers**. These are people who seek out private schools *only when they don't receive the voucher* (otherwise would have gone to public) and vice-versa.

[1] Another term for the LATE you may encounter is the Causal Average Complier Effect (CACE).

Wrap-up

Goals

1. Conduct and interpret RD analysis in simplified data

2. Assess the basic assumptions of the RD design

3. Describe the conceptual and simple mathematical approach for identifying causal effects using the instrumental variables approach

To-Dos

Week 6: Instrumental variables

Readings:

- Murnane & Willett (2011), *MM* Chapters 10-11
- Dee (2004) Are there civic returns to education?
- Angrist et al. (2016) Effects of Boston charter schools
- Further, *MHE*: Ch. 4; *'Metrics*: Ch. 3, *Mixtape*: Ch. 7

Assignments Due:

- DARE #3: 2/18, 11:59pm

Feedback

Plus/Deltas

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